

Minimal Doubling Fermion and Hermiticity

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Abstract

The one-loop Wilsonian renormalization group flows of the two dimensional two-flavors Gross-Neveu model with minimal doubling fermion are calculated numerically. The off-diagonal mass components which are non-Hermiticity are generated by the renormalization group flows. We consider a relation among γ_5 -Hermiticity, R-Hermiticity and PT symmetry. We point out that satisfied two of the three conditions is a sufficient condition for satisfied another condition but not a necessary condition, this relation has a connection with Hermiticity. In the case of periodic and continuum kinetic term, both γ_5 -Hermiticity and R-Hermiticity should be satisfied for consistency.

Because kinetic terms for the minimal doubling fermion do not have R-Hermiticity, non-Hermiticity effective masses appear.

1 Introduction

Lattice gauge theory is one of powerful tools to reveal non-perturbative quark dynamics [1]. In this formulation, space-time coordinate is discretized and physical variables are defined at sites and links. We can calculate observables in strong coupling region using some techniques, for example high temperature expansion. Especially Monte-Carlo simulations are effective methods to investigate non-perturbative physics and carried out actively.

As well known, the doubling problem that fermion has redundant degrees of freedom called "doubblers" is a main problem to apply lattice formulation to theory involved matter fields, for example quantum color dynamics (chromodynamics) (QCD). It is impossible to remove doublers unless we give up some symmetries or properties because of the no-go theorem of Nilsen and Ninomiya [3]. To overcome this problem, many fermion formulations are constructed, for example Wilson fermion [1] and KS fermion [2]. The exact chiral symmetric action is important symmetry to analyze non-perturbative QCD, however this symmetry is incompatible with removing doublers.

Recent year, Creutz constructed exact chiral symmetric lattice fermion action [4], and Borici fitted it to orthogonal lattice [5]. Few decades ago Karsten constructed a fermion formulation which has same structure and is different form from Creutz action [6]. These fermions are called "minimal doubling fermion". The minimal doubling fermion actions break (hyper-)cubic symmetry and some discrete symmetries, such as charge conjugation(C), parity transformation(P), time reflection(T) and so on. Many properties of the fermions are analyzed [6]-[15]¹.

In this paper, we apply minimal doubling fermions to the two dimensional N-flavors Gross-Neveu model which is a toy model of QCD. We calculate the one-loop Wilsonian renormalization group flows (RGFs) in this model.

As a result, the off-diagonal mass components which are non-Hermiticity are generated in this flows, even then starting from massless initial conditions. From this simple analysis, we point out that minimal doubling fermions have a serious defect, non-Hermiticity. Then we discuss the relation among R-Hermiticity, which is defined in Section 4, PT (or C) symmetry and γ_5 Hermiticity.

And we also suggest a criterion to forbid this phenomenon.

This paper is constructed as follows. In Section 2 we will describe for minimal doubling fermion action in two dimension. In Section 3 we will briefly review N-flavors Gross-Neveu model, and then numerically analyze one-loop Wilsonian renormalization group flows around trivial fixed point using naive action and minimal doubling actions. In Section 4 we will discuss Hermiticity, PT symmetry and γ_5 Hermiticity conditions for a general lattice fermion kinetic term. In Section 5 we will conclude and discuss about this relation and minimal doubling fermion.

2 Minimal Doubling Fermion Action

In this section, we will review minimal doubling fermion actions briefly [4]-[17].

We define kinetic terms of naive action(NA) and two minimal doubling actions(MDAs) in two dimensional momentum space as follows:

$$S_{\text{kin}} = \int \frac{d^2p}{(2\pi)^2} \bar{\psi}(-p) D(p) \psi(p), \quad (2.1)$$

¹See ref.[16, 17] about properties and symmetries of hyperdiamond lattice minimal doubling fermion.

where the subscript "kin" means a kinetic term, and

$$D(p) = \begin{cases} \sum_{\mu=1,4} i \sin p_{\mu} \gamma_{\mu} & \equiv D_n(p) \\ i(\sin p_1 + \cos p_4 - 1)\gamma_1 + i(\sin p_4 + \cos p_1 - 1)\gamma_4 & \equiv D_{\text{md1}}(p) \\ i(\sin p_1 + \cos p_4 - 1)\gamma_1 + i \sin p_4 \gamma_4 & \equiv D_{\text{md2}}(p) \end{cases} \quad (2.2)$$

We fix a value of lattice space, $a = 1$, from now on. The subscripts 1, 4 means space and time components respectively. The $D_{\text{md1}}(p)$ and D_{md2} are called "twisted ordering action" and "dropped twisted ordering action" respectively [13]. In two dimension, the NA has four zero-modes and the MDAs have two ones which appear in the following momenta:

$$\begin{aligned} D_n &: \tilde{p} = (0, 0), (0, \pi), (\pi, 0) \text{ and } (\pi, \pi), \\ D_{\text{md1}} &: \tilde{p} = (0, 0) \text{ and } (\pi/2, \pi/2), \\ D_{\text{md2}} &: \tilde{p} = (0, 0) \text{ and } (0, \pi). \end{aligned} \quad (2.3)$$

The "doubblers" appear around each zero-modes as $D(p) = D(\tilde{p} + q)$ with $D(\tilde{p}) = 0$, and they contribute observables.

In the case of NA, the half doublers that half doublers have same chirality and the others have opposite.

In cases of MDAs, they have opposite chirality each other. The NA and MDAs have γ_5 Hermiticity:

$$\gamma_5 D(p) \gamma_5 = D^\dagger(p). \quad (2.4)$$

For massless, they also have chiral symmetry:

$$\gamma_5 D(p) + D(p) \gamma_5 = 0. \quad (2.5)$$

The MDAs violate (hyper-)cubic symmetry and some discrete symmetries. We define charge conjugation(C), parity transformation(P), time reflection(T) and their combinational transformation laws acting on a fermion kinetic term²:

$$\begin{aligned} C &: D(p) \rightarrow -D(-p) \\ P &: D(p) \rightarrow \gamma_4 D(-p_1, p_4) \gamma_4 \\ T &: D(p) \rightarrow -\gamma_4 D(p_1, -p_4) \gamma_4 \\ CP &: D(p) \rightarrow -\gamma_4 D(p_1, -p_4) \gamma_4 \\ CT &: D(p) \rightarrow \gamma_4 D(-p_1, p_4) \gamma_4 \\ PT &: D(p) \rightarrow -D(-p) \\ CPT &: D(p) \rightarrow D(p) \end{aligned} \quad (2.6)$$

We present these symmetric properties of the NA and MDAs in Tab 1³.

3 N-flavors Gross-Neveu model and Renormalization Group Flow in Two Dimension

In this section, we describe the N-flavors Gross-Neveu(GN) model [20] and calculate Wilsonian renormalization group flows(RGFs) using the NA and MDAs numerically. Firstly we will review the N-flavors GN model and then we will calculate the RGFs.

²We can apply same laws to four dimensional theory, replacing with $p_1 \rightarrow \mathbf{p}$.

³"T" represents site and link reflection in lattice space. In the case of the D_{md2} , it has link reflection positive but not site reflection positive [8].

Table 1: Discrete symmetry for the NA and the MDAs

	C	P	T	CP	CT	PT	CPT
naive	○	○	○	○	○	○	○
md1	×	×	×	×	×	×	○
md2	×	×	○	○	×	×	○

3.1 Action and Symmetry

We define the continuum Euclidian Lagrangian of N-flavors GN model as follows:

$$\mathcal{L}_{\text{GN}} = \bar{\psi}(\partial \cdot \gamma + m)\psi - \frac{g^2}{2N}(\bar{\psi}\psi)^2, \quad (3.1)$$

where m is a fermion mass and g is a coupling constant of four fermi interaction. We omit flavor indices if we do not have to write explicitly, $\bar{\psi}\psi \equiv \sum_{i=1}^N \bar{\psi}_i\psi_i$, where "i" means flavor degrees of freedom.

This Lagrangian has $U(1)$ symmetry:

$$\begin{aligned} \psi &\rightarrow e^{i\theta}\psi, \\ \bar{\psi} &\rightarrow \bar{\psi}e^{-i\theta}. \end{aligned} \quad (3.2)$$

In the case of massless fermions, this Lagrangian has chiral \mathbf{Z}_4 symmetry:

$$\begin{aligned} \psi &\rightarrow (i\gamma_5)^n \psi, \\ \bar{\psi} &\rightarrow \bar{\psi} (i\gamma_5)^n. \end{aligned} \quad (n = 0, 1, 2, 3) \quad (3.3)$$

In the case of massive fermions, chiral \mathbf{Z}_4 symmetry reduces to chiral \mathbf{Z}_2 symmetry ($n = 0, 2$). In addition, if all flavors have same masses it has the $SU(N)_F$ symmetry:

$$\begin{aligned} \psi_i &\rightarrow U_{ij}\psi_j, \\ \bar{\psi}_i &\rightarrow \bar{\psi}_j U_{ji}^\dagger, \end{aligned} \quad (U \in SU(N)) \quad (3.4)$$

It is convenient to redefine the GN action using an auxiliary scalar field σ instead of $(\bar{\psi}\psi)$:

$$\mathcal{L}_{\text{GN}} = \bar{\psi}(\partial \cdot \gamma + m)\psi + \frac{N}{2}\sigma^2 + g\sigma\bar{\psi}\psi. \quad (3.5)$$

According to this manipulation, we can obtain the action which involves Yukawa interaction instead of four fermi interaction. According to perterbative calculation, the NG model has asymptotic freedom property [21, 22].

3.2 Wilsonian Renormalization Group Flow

Using the Wilsonian method [23], we calculate numerically the RGFs for the mass and coupling constant starting from the trivial fixed point ($m = g^2 = 0$).

We explain how to calculate Wilsonian RGFs in Appendix A.

In the case of MDAs, we use each zero-modes as the different flavor fermions, and in the case of NA we use only two zero-modes, $\tilde{p} = (0, 0)$ and (π, π) . We now represent the spinor indices explicitly, and we distinguish 0, 1 from 2, 3 as different flavors. We assume that high frequency modes of fields are not effective, therefore we neglect their contributions, $\psi(1 < |k|)$, $\bar{\psi}(1 < |k|)$ and $\sigma(1 < |k|)$.

We choose initial conditions for the mass as $m_{00} = m_{11} = m_{22} = m_{33} = 0, \pm 0.25, \pm 0.5$, and for the coupling constant as $g^2 = 0, 0.2, 0.4$.

The off-diagonal mass components equal to zero in all cases. We will calculate numerically up to one-loop quantum effects and RGFs which toward IR from the initial conditions⁴. In our calculation we define γ -matrices as follows:

$$\gamma_1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (3.6)$$

The results are referred in fig.1. The RGFs of the NA and MDAs are similar forms and differences among each values are $O(10^{-3})$.

In cases of MDAs, however, off-diagonal mass components are generated by the RGFs, except the initial value which is trivial fixed point ($m = 0, g^2 = 0$). The obtained results do not have Hermiticity.

We show this fact in fig.2 and fig.3 whose initial conditions are $m = 0, g^2 = 0.2$. The fig.2(a) and (b) are the RGFs of $D_{\text{md1}}(p)$ and $D_{\text{md2}}(p)$ respectively. The fig.3(a) and (b) are relations between the off-diagonal mass components and iterations using D_{md1} and D_{md2} respectively. Though the off-diagonal mass components amplify as the flows go to IR because of scaling effect, they do not break chiral \mathbf{Z}_4 symmetry. This fact means that the MDAs do not have Hermiticity and it seems to have any problems. In the following section, we will discuss this phenomenon and Hermiticity.

4 γ_5 -Hermiticity, R-Hermiticity and PT symmetry

In the previous section, we observed non-Hermiticity contributions in fermion effective mass components. In Euclidian formulation, a fermion kinetic term in continuum limit does not have Hermiticity but anti-Hermiticity. The NA also has an anti-Hermiticity kinetic term. In the case of NA, however, an effective mass does not have anti-Hermite components explicitly, therefore the cause of them is clearly added even function terms, $(\cos p - 1)\gamma$. We will consider Hermiticity and PT symmetry, and what added terms generate explicit non-Hermite effective coupling constants. Hermiticity is discussed in ref.[18] at the case of the minimal doubling fermion on the Hyperdiamond lattice. We will focus on only kinetic terms from now on and discuss In the case of two dimension⁵.

We will focus on only kinetic terms from now on. In the case of two dimension, we define a translation invariant kinetic term in momentum space as follows:

$$S = \int \frac{d^2 k}{(2\pi)^2} \bar{\psi}(-k) D(k) \psi(k), \quad (4.1)$$

with

$$D(k) = \sum_{\mu=1,4} f_{\mu}(k) \gamma_{\mu}, \quad (4.2)$$

where $f_{\mu}(k)$ are complex numbers in general. We define three conditions, γ_5 -Hermiticity, R-Hermiticity and PT symmetry⁶:

$$\gamma_5 \text{Hermiticity} : D(k) = \gamma_5 D^{\dagger}(k) \gamma_5, \quad (4.3)$$

$$\text{R-Hermiticity} : D(k) = D^{\dagger}(-k), \quad (4.4)$$

$$\text{PT symmetry} : D(k) = \gamma_5 D(-k) \gamma_5. \quad (4.5)$$

⁴According to estimate integrating part of one-loop calculation we used the sectional measurement method. We took the length of division $\Delta p_{\mu} = 0.01$ ($O(0.01^2)$ error in the integrating part and it seems that accuracy is not very well.

⁵We can apply this discussion to the case of any even dimension.

⁶In following discussion, we can use C symmetry instead of PT symmetry because of CPT theorem.

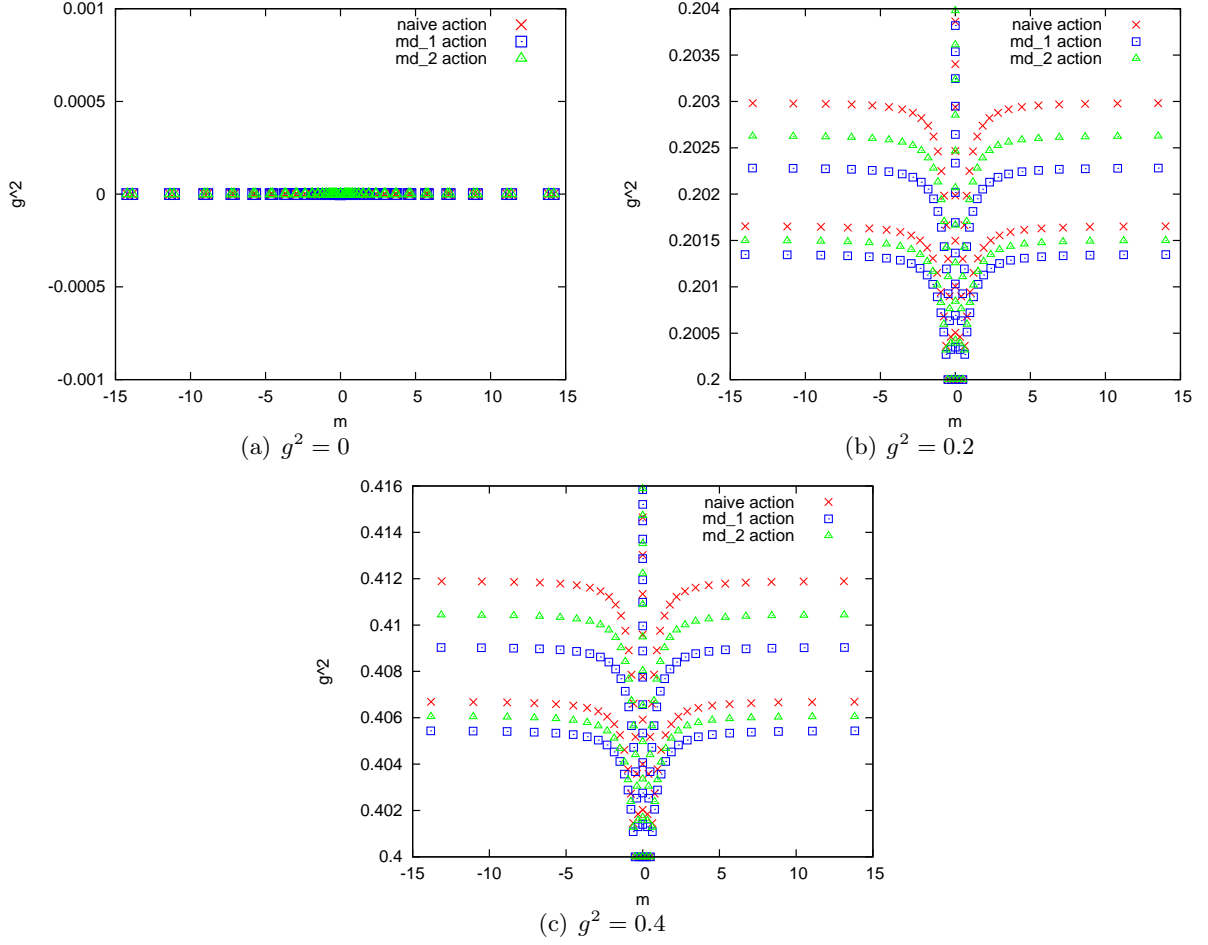


Figure 1: The RGFs of MA and MDAs. Initial parameters are $m_{11} = m_{22} = m_{33} = m_{44} = 0, \pm 0.25, \pm 0.5$, $g^2 = (a)0, (b)0.2, (c)0.4$. The RGFs toward IR which have large mass.

These conditions are not independent of one another. We can easily lead to the fact that another condition is automatically satisfied if a kinetic term satisfy two of the three conditions. This is a sufficient condition but not a necessary condition. If $f_\mu(k)$ is pure imaginary, γ_5 -Hermiticity guarantees anti-Hermite condition for a kinetic term. R-Hermiticity is also used as (anti-)Hermite condition, for example in ref.[19], however it is not well-defined because forward-derivative, $D_{fd}(k) = \sum_\mu (e^{ik_\mu} - 1) \gamma_\mu$, satisfy this condition.

We will show that R-Hermiticity is a condition for real effective coupling constants. We assume that a fermion kinetic term has R-Hermiticity and effective coupling constants is the following form:

$$g_{eff} = g_0 + \sum_{n=1}^{\infty} I^{(n)}, \quad (4.6)$$

with

$$I^{(n)} = \int \prod_{i=1}^r \frac{d^2 k_i}{(2\pi)^2} \cdot I_{\alpha_1 \beta_1 \dots \alpha_r \beta_r}^{(n)}(-k_1, \dots, -k_r) \cdot \prod_{j=1}^r S_{\alpha_j \beta_j}(k_j), \quad (4.7)$$

where g_0 is a real bare coupling constant, g_{eff} is an effective coupling constant, $S_{\alpha\beta}(k)$ is a fermion propagator and $I^{(n)}$ is n -loop quantum effect which is constructed from r -fermion propagators. We

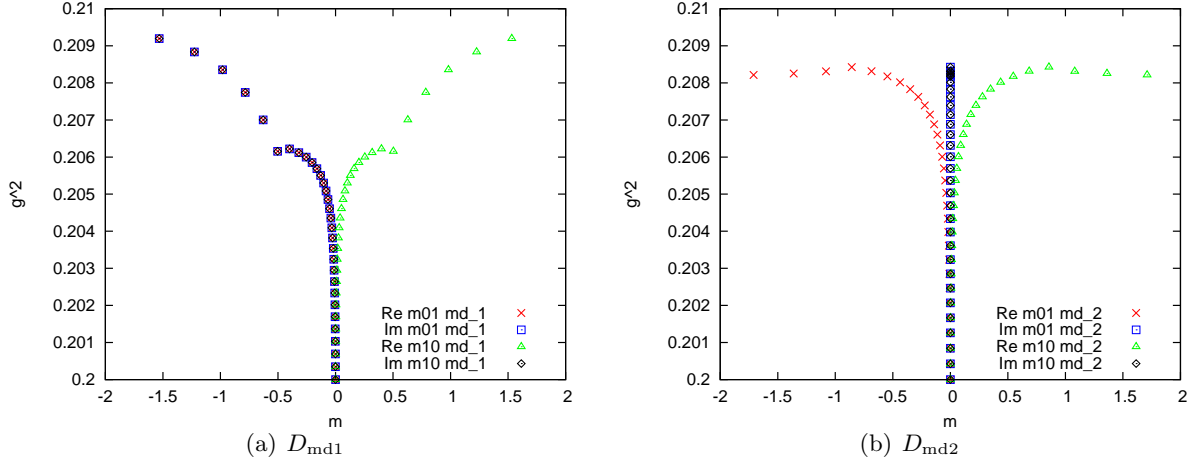


Figure 2: The off-diagonal mass and coupling constant of the MDAs, (a) D_{md1} , (b) D_{md2} . Initial parameters are $m_{11} = m_{22} = m_{33} = m_{44} = 0, g^2 = 0.2$. The RGFs toward IR which has large mass. The RGFs have irregular forms and the off-diagonal mass components are non-Hermiticity.

act Hermite conjugate to the second term of r.h.s. The effective parameter is real if the following condition is satisfied,

$$I_{\alpha_1\beta_1\cdots\alpha_r\beta_r}^{(n)}(-k_1, \cdots, -k_r) = I_{\alpha_r\beta_r\cdots\alpha_1\beta_1}^{(n)\dagger}(k_1, \cdots, k_r). \quad (4.8)$$

Ordinary Euclidian actions are constructed from Hermite terms except a fermion kinetic term, so that $I^{(n)}$ is also constructed from only Hermite ingredients. Therefore we can use R-Hermiticity as a reality condition for parameters, as long as (4.8) is satisfied. If γ_5 -Hermiticity is not satisfied and R-Hermiticity is satisfied, one-loop fermion propagator produces diagonal mass components. In the continuum Euclidian action case, these components are not produced. Therefore both γ_5 -Hermiticity and R-Hermiticity should be satisfied for consistency.

Next, we will show that PT symmetry is always broken if we add extra kinetic terms to a NA to reduce to doublers⁷.

Statement

In even dimension, a PT symmetric kinetic term which is assumed periodicity and continuity always has more than or equals to 2^d poles.

Proof

For simplicity, we also assume translation invariant⁸. A general 2π periodic and continuum $D(k)$ is the following form:

$$D(k) = \sum_{\mu, \nu=1}^d \sum_{n \in \mathbf{N}^d}^{\infty} [(A_{\mu}(n) + iB_{\mu}(n)) \cos(n_{\nu}k_{\nu}) + (C_{\mu}(n) + iD_{\mu}(n)) \sin(n_{\nu}k_{\nu}) + E_{\mu}] \gamma_{\mu}, \quad (4.9)$$

where $A_{\mu}(n), B_{\mu}(n), C_{\mu}(n), D_{\mu}(n)$ are real numbers and E_{μ} are complex constants. From PT symmetry,

$$A_{\mu}(n) = B_{\mu}(n) = E_{\mu} = 0, \quad \text{for all } \mu, n. \quad (4.10)$$

⁷This argument has discussed in ref.[9] but not mathematically.

⁸We can similarly lead to same statement without translation invariance. In the case of non-translation invariant, a kinetic term is $D(k, p)$ form and at least 4^d doublers appear.

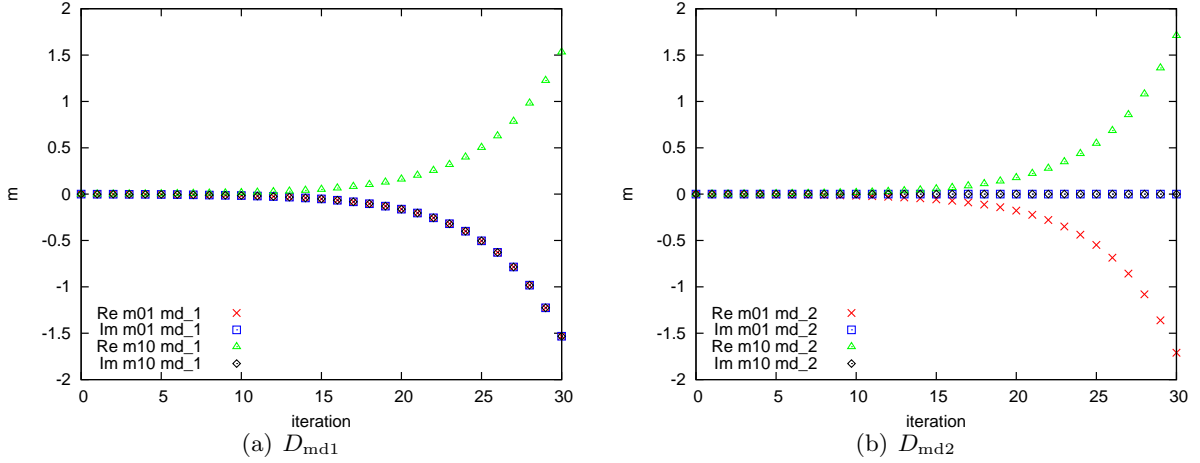


Figure 3: The off-diagonal mass and iteration of the MDAs, (a) D_{md1} , (b) D_{md2} . Initial parameters are $m_{11} = m_{22} = m_{33} = m_{44} = 0, g^2 = 0.2$. The off-diagonal mass components are generated and they have non-Hermiticity.

The $D(k)$ always has two poles at $k = 0$ and π for each dimension.
Therefore $D(k)$ has equal or more than 2^d poles.

This statement equals to that we can not reduce to the number of doublers using PT symmetric kinetic terms⁹. In numerical simulation context, γ_5 -Hermiticity is a very important condition to avoid sign problem. Assumed translation invariance, R-Hermiticity is not satisfied if $D(k)$ satisfy γ_5 Hermiticity and does not PT symmetry. Therefore effective parameters have explicit non-Hermiticity. In a process of rewriting from Minkovskian to Euclidian, Hermite fermion kinetic terms transmute anti-Hermite ones. A general lattice fermions which is reduced the number of doublers have possibilities of generating this anti-Hermite effective coupling constants. The R-Hermiticity is a criterion to remove non-Hermiticity. Because MDAs have only γ_5 -Hermiticity, the fermion effective mass which is constructed from odd fermion propagators has non-Hermiticity components explicitly in RGFs.

5 Conclusion and Discussion

We have investigated the one-loop Wilsonian renormalization group flows(RGFs) of the two-flavors Gross-Neveu(GN) model in two dimension with naive action(NA) and two minimal doubling actions(MDAs). We observed that in cases of MDAs off-diagonal mass components which are non-Hermiticity are generated even massless initial conditions. In order to understand the reason for non-Hermiticity mass components, we considered γ_5 Hermiticity, R-Hermiticity and PT (or C) symmetry conditions. These conditions are not independent, satisfied two of the three conditions is a sufficient condition for that another condition is satisfied automatically. However it is not a necessary condition. We suggested that R-Hermiticity is a condition for removing non-Hermiticity. However, both γ_5 -Hermiticity and R-Hermiticity should be satisfied. Because the diagonal mass components which is not consistent are generated if only R-Hermiticity is satisfied. Therefore we can not reduce the numbers of doubler with modified kinetic terms which have periodicity and continuity because of a relation between doublers and PT symmetry.

⁹We can not apply this statement to non- γ_μ linear case, for example Wilson fermion.

We applied this relation to the MDAs which have only γ_5 -Hermicity. Because of the non-Hermite kinetic terms, explicit non-Hermite effective mass components appear.

In lattice perturbative theory we have to care whether we can remove relevant and marginal symmetry breaking terms with counter-terms¹⁰. In massive two-flavors GN model, non-Hermite masses are small in comparison with diagonal masses and we can remove them with counter-terms. In massless case, non-Hermite masses appear however they do not break chiral \mathbf{Z}_4 symmetry. Therefore it is not so trouble at one-loop level. In more higher loop or non-perturbative case, we do not know how far parameters are involved the non-Hermite effects and should be fine-tuned.

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¹⁰This issues are discussed in the case of four dimensional gauge theory in ref.[11] for example.

Appendix

A Wilsonian Renormalization Group

In this appendix, we review a method to calculate the Wilsonian renormalization group flow in the case of GN model in two dimension [23].

We define the partition function of the GN model in momentum space ¹¹:

$$Z = \int D\sigma D\psi D\bar{\psi} \exp(-S_{\text{GN}}), \quad (\text{A.1})$$

with

$$S_{\text{GN}} = \int_{0 < |p| < 1} \frac{d^2 p}{(2\pi)^2} \mathcal{L}_{\text{GN}}, \quad (\text{A.2})$$

where

$$D\sigma = \prod_{0 < |k| < 1} d\sigma(k), \quad D\psi = \prod_{0 < |k| < 1} d\psi(k), \quad D\bar{\psi} = \prod_{0 < |k| < 1} d\bar{\psi}(k), \quad (\text{A.3})$$

and \mathcal{L}_{GN} is given in (3.5). We can treat of N as a mass parameter of auxiliary field σ . Here we assumed that the high frequency modes had already integrated and they do not contribute effectively. Then we split the field configurations as follows:

$$\sigma(p) = \sigma_l(p) + \sigma_h(p), \quad (\text{A.4})$$

where

$$\sigma_l(p) = \sigma(p) \quad \text{if } 0 < |p| < \frac{4}{5}, \quad \text{zero otherwise}, \quad (\text{A.5})$$

$$\sigma_h(p) = \sigma(p) \quad \text{if } \frac{4}{5} < |p| < 1, \quad \text{zero otherwise}, \quad (\text{A.6})$$

and the other fields are also split similarly ¹². We choice renormalization conditions as follows:

$$\Gamma_{\psi}^{(2)}(0, 0) = -m_R, \quad (\text{A.7})$$

$$\Gamma_{\sigma}^{(2)}(0, 0) = -N_R, \quad (\text{A.8})$$

$$\Gamma^{(3)}(0, 0, 0) = -g_R, \quad (\text{A.9})$$

where $\Gamma^{(i)}$ are renormalized i -point functions, m_R, N_R, g_R are renormalized parameters and arguments of $\Gamma^{(i)}$ are external momenta. In order to obtain effective parameters, we calculate one-loop effect and integrate out only high frequency modes:

$$m_{R\alpha\beta} = \left(\frac{5}{4}\right)^{-2} \eta_{\psi}^2 \left[m - g^2 \int_{\frac{4}{5} < |k| < 1} \frac{d^2 k}{(2\pi)^2} S_{\alpha\beta}(k) D(k) \right], \quad (\text{A.10})$$

$$\frac{N_R}{2} = \left(\frac{5}{4}\right)^{-2} \eta_{\sigma}^2 \left[\frac{N}{2} + \frac{g^2}{2} \int_{\frac{4}{5} < |k| < 1} \frac{d^2 k}{(2\pi)^2} \text{tr} [S(k) S(k)] \right], \quad (\text{A.11})$$

$$g_R = \left(\frac{5}{4}\right)^{-4} \eta_{\psi}^2 \eta_{\sigma} \left[g + g^3 \int_{\frac{4}{5} < |k| < 1} \frac{d^2 k}{(2\pi)^2} (S(k) S(k))_{\alpha\beta} D(k) \right] \cdot \delta_{\alpha\beta}, \quad (\text{A.12})$$

¹¹We omit the subscript which means flavors.

¹²On account of numerical efficiency, we choose a division which split between σ_l and σ_h as $p = \frac{4}{5}$.

where $S(k)$ and $D(k)$ are propagators of each fields presented below, "tr" is a trace operation of the fermionic indices and η_ψ and η_σ are rescaling parameters of fermion and auxiliary field respectively. We can define these parameters with dimensional analysis in the following values:

$$\eta_\psi = \left(\frac{5}{4}\right)^{3/2}, \quad (\text{A.13})$$

$$\eta_\sigma = \frac{5}{4}. \quad (\text{A.14})$$

We can obtain propagators from the GN action:

$$S(k) = \left[D_f(\tilde{k} + k) + m \right]^{-1}, \quad (\text{A.15})$$

$$D(k) = \frac{1}{N}, \quad (\text{A.16})$$

where $D_f(k)$ is one of the lattice fermion kinetic terms in (2.2) and \tilde{k} is a zero-mode momentum in (2.4). Substituting (A.15) and (A.16) to (A.11)-(A.12), we can obtain effective mass and coupling constant after integrating out.

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